

Numbers, Variables, and Mr. Russell's Philosophy. By P. RICHARDSON and E. H. LANDIS. Open Court Co. Pp. 59.

This little book is reprinted from *The Monist* of July, 1915, and is bound up with the preface to a forthcoming work by the same authors in thirteen parts called *Fundamental Conceptions of Modern Mathematics*. The first part of this has been received and will be noticed in due course.

The present work criticises Mr. Russell's theories of mathematics entirely with reference to *The Principles of Mathematics*; nothing is said of the *Principia* or of logical articles in *MIND* and other places which have appeared since 1903 and shown modifications in Mr. Russell's views.

The first criticism is that Russell makes all mathematical arguments to be of the form A, B, C , etc., imply T ; whilst mathematics asserts its premises and so asserts its conclusions. This is hardly correct. If it be true that pure mathematics can be reduced to logic the ultimate premises of pure mathematics will be the axioms of logic. These are asserted by logic, and therefore anything that can be formally deduced from them is also asserted by logic.

The most severe criticisms that are passed in this book are on Russell's theory of number. He is held to be wrong in thinking that two classes can have the same number; if the classes differ they only have equal numbers. In consequence of this he is blamed for regarding such symbols as 2 as proper names, and such sentences as $2 + 2 = 4$ as standing for singular propositions. According to the authors there are as many different 2's as there are couples and the symbol 2 is a general name for these, and arithmetical propositions are universal and not singular. This $2 \times 3 = 6$ means every product of a two by a three is either equal to or identical with any six. The point of the disjunction is that the product will only be equal to a six which belongs to a class other than that formed from the two particular classes to which the two and the three in any given case belong.

Now this theory of the authors cannot, I think, be refuted. It might be true; but it is more complicated than Russell's, and the arguments which they produce for it seem to me quite worthless. They urge that two different objects cannot have the same attribute but only similar attributes; identity of attribute only refers to a single object kept under continuous observation. No ground is produced for this opinion except the authority of Mr. Spencer with whom Russell is presumed to be unacquainted! Equality then is perfect likeness between the number attributes of several classes. I am certainly not impressed with the argument that it is as foolish e.g. to call the number of my eyes the same as the number of my ears as it would be to call two precisely similar houses the same house. Yet this seems to be the main argument which the authors use.

It is further objected that the definition of numbers as classes of similar classes is circular; 'it is like defining whiteness as the class of all white objects'. But it is not. Similarity of classes is defined without any reference to number, whilst white cannot be analysed. If you could show that the statement *A and B and . . . are white* is equivalent to *A and B and . . . have the relation R to each other*, when R does not involve the notion of white, there would be no circularity in defining whiteness as the class of all white objects.

Russell's definition of quantity and his distinction between quantity and magnitude are adversely criticised. The authors do not accept the argument from the Principle of Abstraction for the absolute theory of

magnitude; and here we may sympathise with them. They themselves use quantity to cover (a) Russell's quantities (*e.g.* foot-rules, pounds of butter, etc.); (b) Russell's magnitudes (*e.g.* 2 feet, 1 pound); and (c) abstract numbers.

Variables we are told are not quantities; nor are they mere symbols; they are classes of quantities in which it is the *relations* between the quantities and not their other properties which are important. Russell is blamed for his attempt to extend the notion of variables to cases where there is no reference to quantity and for his attempt to associate them with the notion of *any* as distinct from *every*. I am inclined to think that there is a distinction between any *u* and every *u*, though it is hard to bring it out. It is certainly an unfair criticism of Russell to say that the opposition in which he puts *any* and *every* implies that what is about any number is not about every number. *Any* and *every* might be different concepts and yet what is true of any *u* may necessarily be true of every *u*.

It is contended that Russell got his notion of variables by considering logical and mathematical identities like $(x + y)(x - y) = x^2 - y^2$. But if you take the equation $x + y = 10$ you cannot interpret this to mean: Any number added to any number is equal to 10; so that Russell's theory of variables will not apply to these cases. The authors regard such cases not as examples of variables; the *x*'s and *y*'s are just class names for classes of numbers and the equation expresses a functional relation between *corresponding* numbers of these classes. The true account, however, surely is that both equations are *proportional functions*; that in each the variables are only restricted by considerations of type; but that the former gives true *propositions* for all values of *x* and *y*, whilst the latter only does so for *some* values.

A word of praise is due to the authors for pointing out many places in the *Principles of Mathematics* where Russell is far from clear as to whether he is talking of verbal expressions or the objects that they stand for. But they would have found most of their criticisms answered by anticipation if they had studied Russell's later writings.

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Authority, Liberty, and Function in the Light of the War: a Critique of Authority and Liberty as the Foundations of the Modern State, and an attempt to base Societies on the Principle of Function. By RAMIRO DE MAEZTU. London: Unwin & Allen. Cr. 8vo., pp. 288. 4s. 6d. net.

The contents of this book have appeared as articles in the *New Age*, but they were intended for a complete work, and they form in fact a very coherent treatise. The author's positive thesis is the desirability of ordering society on a "functional" system, meaning that rights and claims are to depend on the discharge of function, which = the production of values. From this point of view he advocates Syndicalism, and at the same time the system known as that of National Guilds. In the main point which interests him they coincide, and he does not, I think, take trouble to distinguish between them.

His point then is that the only just rights and laws are "objective," as opposed to "subjective" rights or rights attaching to mere "personality," which are for him the enemy. Objective rights are based on function; subjective rights are free and arbitrary (p. 50). The latter, as I gathered, may be anything from the old-fashioned "natural" right